CHAPTER

14

ALGEBRAIC FRACTIONS, AND EQUATIONS AND INEQUALITIES INVOLVING FRACTIONS

Although people today are making greater use of decimal fractions as they work with calculators, computers, and the metric system, common fractions still surround us.

We use common fractions in everyday measures: $\frac{1}{4}$-inch nail, $2\frac{1}{2}$-yard gain in football, $\frac{1}{2}$ pint of cream, $1\frac{1}{3}$ cups of flour. We buy $\frac{1}{2}$ dozen eggs, not 0.5 dozen eggs. We describe 15 minutes as $\frac{1}{4}$ hour, not 0.25 hour. Items are sold at a third $\left(\frac{1}{3}\right)$ off, or at a fraction of the original price.

Fractions are also used when sharing. For example, Andrea designed some beautiful Ukrainian eggs this year. She gave one-fifth of the eggs to her grandparents. Then she gave one-fourth of the eggs she had left to her parents. Next, she presented her aunt with one-third of the eggs that remained. Finally, she gave one-half of the eggs she had left to her brother, and she kept six eggs. Can you use some problem-solving skills to discover how many Ukrainian eggs Andrea designed?

In this chapter, you will learn operations with algebraic fractions and methods to solve equations and inequalities that involve fractions.

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14-1 THE MEANING OF AN ALGEBRAIC FRACTION

A *fraction* is a quotient of any number divided by any nonzero number. For example, the arithmetic fraction $\frac{3}{4}$ indicates the quotient of 3 divided by 4.

An **algebraic fraction** is a quotient of two algebraic expressions. An algebraic fraction that is the quotient of two polynomials is called a **fractional expression** or a **rational expression**. Here are some examples of algebraic fractions that are rational expressions:

$$\frac{x}{2} \qquad \frac{2}{x} \qquad \frac{4c}{3d} \qquad \frac{x+5}{x-2} \qquad \frac{x^2+4x+3}{x+1}$$

The fraction $\frac{a}{b}$ means that the number represented by a, the numerator, is to be divided by the number represented by b, the denominator. Since division by 0 is not possible, the value of the denominator, b, cannot be 0. An algebraic fraction is defined or has meaning only for values of the variables for which the denominator is not 0.

EXAMPLE 1

Find the value of x for which $\frac{12}{x-9}$ is not defined.

Solution The fraction $\frac{12}{x-9}$ is not defined when the denominator, $x - 9$, is equal to 0.

$$x - 9 = 0$$
$$x = 9 \text{ Answer}$$

EXERCISES

Writing About Mathematics

1. Since any number divided by itself equals 1, the solution set for $\frac{x}{x} = 1$ is the set of all real numbers. Do you agree with this statement? Explain why or why not.

2. Aaron multiplied $\frac{b-1}{1+\frac{1}{b}}$ by $\frac{b}{b}$ (equal to 1) to obtain the fraction $\frac{b^2-b}{b+1}$. Is the fraction $\frac{b-1}{1+\frac{1}{b}}$ equal to the fraction $\frac{b^2-b}{b+1}$ for all values of b? Explain your answer.

Developing Skills

In 3–12, find, in each case, the value of the variable for which the fraction is not defined.

3. $\frac{2}{x}$ **4.** $\frac{-5}{6x}$ **5.** $\frac{12}{y^2}$ **6.** $\frac{1}{x-5}$ **7.** $\frac{7}{2-x}$

8. $\frac{y+5}{y+2}$ **9.** $\frac{10}{2x-1}$ **10.** $\frac{2y+3}{4y+2}$ **11.** $\frac{1}{x^2-4}$ **12.** $\frac{3}{x^2-5x-14}$

Applying Skills

In 13–17, represent the answer to each problem as a fraction.

13. What is the cost of one piece of candy if five pieces cost c cents?

14. What is the cost of 1 meter of lumber if p meters cost 980 cents?

15. If a piece of lumber $10x + 20$ centimeters in length is cut into y pieces of equal length, what is the length of each of the pieces?

16. What fractional part of an hour is m minutes?

17. If the perimeter of a square is $3x + 2y$ inches, what is the length of each side of the square?

14-2 REDUCING FRACTIONS TO LOWEST TERMS

A fraction is said to be **reduced to lowest terms** or is a **lowest terms fraction** when its numerator and denominator have no common factor other than 1 or −1.

Each of the fractions $\frac{5}{10}$ and $\frac{a}{2a}$ can be expressed in lowest terms as $\frac{1}{2}$.

The arithmetic fraction $\frac{5}{10}$ is reduced to lowest terms when both its numerator and denominator are divided by 5:

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

The algebraic fraction $\frac{a}{2a}$ is reduced to lowest terms when both its numerator and denominator are divided by a, where $a \neq 0$:

$$\frac{a}{2a} = \frac{a \div a}{2a \div a} = \frac{1}{2}$$

Fractions that are equal in value are called **equivalent fractions**. Thus, $\frac{5}{10}$ and $\frac{1}{2}$ are equivalent fractions, and both are equivalent to $\frac{a}{2a}$, when $a \neq 0$.

The examples shown above illustrate the **division property of a fraction**: if the numerator and the denominator of a fraction are divided by the same nonzero number, the resulting fraction is equal to the original fraction.

In general, for any numbers a, b, and x, where $b \neq 0$ and $x \neq 0$:

$$\boldsymbol{\frac{ax}{bx} = \frac{ax \div x}{bx \div x} = \frac{a}{b}}$$

When a fraction is reduced to lowest terms, we list the values of the variables that must be excluded so that the original fraction is equivalent to the reduced form and also has meaning. For example:

$$\frac{4x}{5x} = \frac{4x \div x}{5x \div x} = \frac{4}{5} \text{ (where } x \neq 0\text{)}$$

$$\frac{cy}{dy} = \frac{cy \div y}{dy \div y} = \frac{c}{d} \text{ (where } y \neq 0, d \neq 0\text{)}$$

When reducing a fraction, the division of the numerator and the denominator by a common factor may be indicated by a **cancellation**.

Here, we use cancellation to divide the numerator and the denominator by 3:

$$\frac{3(x+5)}{18} = \frac{\overset{1}{\cancel{3}}(x+5)}{\underset{6}{\cancel{18}}} = \frac{x+5}{6}$$

Here, we use cancellation to divide the numerator and the denominator by $(a - 3)$:

$$\frac{a^2-9}{3a-9} = \frac{\overset{1}{\cancel{(a-3)}}(a+3)}{3\underset{1}{\cancel{(a-3)}}} = \frac{a+3}{3}$$

(where $a \neq 3$)

By re-examining one of the examples just seen, we can show that the *multiplication property of one* is used whenever a fraction is reduced:

$$\frac{3(x+5)}{18} = \frac{3 \cdot (x+5)}{3 \cdot 6} = \frac{3}{3} \cdot \frac{(x+5)}{6} = 1 \cdot \frac{(x+5)}{6} = \frac{x+5}{6}$$

However, when the multiplication property of one is applied to fractions, it is referred to as the **multiplication property of a fraction**. In general, for any numbers a, b, and x, where $b \neq 0$ and $x \neq 0$:

$$\frac{a}{b} = \frac{a}{b} \cdot \frac{x}{x} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

Procedure

To reduce a fraction to lowest terms:

METHOD 1

1. Factor completely both the numerator and the denominator.
2. Determine the greatest common factor of the numerator and the denominator.
3. Express the given fraction as the product of two fractions, one of which has as its numerator and its denominator the greatest common factor determined in step 2.
4. Use the multiplication property of a fraction.

METHOD 2

1. Factor both the numerator and the denominator.
2. Divide both the numerator and the denominator by their greatest common factor.

EXAMPLE 1

Reduce $\frac{15x^2}{35x^4}$ to lowest terms.

Solution **METHOD 1**

$$\frac{15x^2}{35x^4} = \frac{3}{7x^2} \cdot \frac{5x^2}{5x^2}$$
$$= \frac{3}{7x^2} \cdot 1$$
$$= \frac{3}{7x^2}$$

METHOD 2

$$\frac{15x^2}{35x^4} = \frac{3 \cdot 5x^2}{7x^2 \cdot 5x^2}$$
$$= \frac{3 \cdot \overset{1}{\cancel{5x^2}}}{7x^2 \cdot \underset{1}{\cancel{5x^2}}} = \frac{3}{7x^2}$$

Answer $\frac{3}{7x^2}$ $(x \neq 0)$

EXAMPLE 2

Express $\frac{2x^2 - 6x}{10x}$ as a lowest terms fraction.

Solution **METHOD 1**

$$\frac{2x^2 - 6x}{10x} = \frac{2x(x-3)}{2x \cdot 5}$$
$$= \frac{2x}{2x} \cdot \frac{(x-3)}{5}$$
$$= 1 \cdot \frac{(x-3)}{5}$$
$$= \frac{x-3}{5}$$

METHOD 2

$$\frac{2x^2 - 6x}{10x} = \frac{2x(x-3)}{10x}$$
$$= \frac{\overset{1}{\cancel{2x}}(x-3)}{\underset{5}{\cancel{10x}}} = \frac{x-3}{5}$$

Answer $\frac{x-3}{5}$ $(x \neq 0)$

EXAMPLE 3

Reduce each fraction to lowest terms.

a. $\frac{x^2 - 16}{x^2 - 5x + 4}$ **b.** $\frac{2 - x}{4x - 8}$

Solution **a.** Use Method 1:

$$\frac{x^2 - 16}{x^2 - 5x + 4} = \frac{(x+4)(x-4)}{(x-1)(x-4)}$$
$$= \frac{x+4}{x-1} \cdot \frac{x-4}{x-4}$$
$$= \frac{x+4}{x-1} \cdot 1$$
$$= \frac{x+4}{x-1}$$

b. Use Method 2:

$$\frac{2-x}{4x-8} = \frac{-1(x-2)}{4(x-2)}$$
$$= \frac{-1\overset{1}{\cancel{(x-2)}}}{4\underset{1}{\cancel{(x-2)}}}$$
$$= -\frac{1}{4}$$

Answers **a.** $\frac{x+4}{x-1}$ $(x \neq 1, x \neq 4)$ **b.** $-\frac{1}{4}$ $(x \neq 2)$

EXERCISES

Writing About Mathematics

1. Kevin used cancellation to reduce $\frac{a+4}{a+8}$ to lowest terms as shown below. What is the error in Kevin's work?

$$\frac{a+4}{a+8} = \frac{\overset{1}{\cancel{a}} + \overset{1}{\cancel{4}}}{\underset{1}{\cancel{a}} + \underset{2}{\cancel{8}}} = \frac{1+1}{1+2} = \frac{2}{3}$$

2. Kevin let $a = 4$ to prove that when reduced to lowest terms, $\frac{a+4}{a+8} = \frac{2}{3}$. Explain to Kevin why his reasoning is incorrect.

Developing Skills

In 3–54, reduce each fraction to lowest terms. In each case, list the values of the variables for which the fractions are not defined.

3. $\frac{4x}{12x}$ **4.** $\frac{27y^2}{36y}$ **5.** $\frac{24c}{36d}$ **6.** $\frac{9r}{10r}$

7. $\frac{ab}{cb}$ **8.** $\frac{3ay^2}{6by^2}$ **9.** $\frac{5xy}{9xy}$ **10.** $\frac{2abc}{4abc}$

11. $\frac{15x^2}{5x}$ **12.** $\frac{5x^2}{25x^4}$ **13.** $\frac{27a}{36a^2}$ **14.** $\frac{8xy^2}{24x^2y}$

15. $\frac{+12a^2b}{-8ac}$ **16.** $\frac{-20x^2y^2}{-90xy^2}$ **17.** $\frac{-32a^3b^3}{+48a^3b^3}$ **18.** $\frac{5xy}{45x^2y^2}$

19. $\frac{3x+6}{4}$ **20.** $\frac{8y-12}{6}$ **21.** $\frac{5x-35}{5x}$ **22.** $\frac{8m^2+40m}{8m}$

23. $\frac{2ax+2bx}{6x^2}$ **24.** $\frac{5a^2-10a}{5a^2}$ **25.** $\frac{12ab-3b^2}{3ab}$ **26.** $\frac{6x^2y+9xy^2}{12xy}$

27. $\frac{18b^2+30b}{9b^3}$ **28.** $\frac{4x}{4x+8}$ **29.** $\frac{7d}{7d+14}$ **30.** $\frac{5y}{5y+5x}$

31. $\frac{2a^2}{6a^2-2ab}$ **32.** $\frac{14}{7r-21s}$ **33.** $\frac{12a+12b}{3a+3b}$ **34.** $\frac{x^2-9}{3x+9}$

35. $\frac{x^2-1}{5x-5}$ **36.** $\frac{1-x}{x-1}$ **37.** $\frac{3-b}{b^2-9}$ **38.** $\frac{2s-2r}{s^2-r^2}$

39. $\frac{16-a^2}{2a-8}$ **40.** $\frac{x^2-y^2}{3y-3x}$ **41.** $\frac{2b(3-b)}{b^2-9}$ **42.** $\frac{r^2-r-6}{3r-9}$

43. $\frac{x^2+7x+12}{x^2-16}$ **44.** $\frac{x^2+x-2}{x^2+4x+4}$ **45.** $\frac{3y-3}{y^2-2y+1}$ **46.** $\frac{x^2-3x}{x^2-4x+3}$

47. $\frac{x^2-25}{x^2-2x-15}$ **48.** $\frac{a^2-a-6}{a^2-9}$ **49.** $\frac{a^2-6a}{a^2-7a+6}$ **50.** $\frac{2x^2-50}{x^2+8x+15}$

51. $\frac{r^2-4r-5}{r^2-2r-15}$ **52.** $\frac{48+8x-x^2}{x^2+x-12}$ **53.** $\frac{2x^2-7x+3}{(x-3)^2}$ **54.** $\frac{x^2-7xy+12y^2}{x^2+xy-20y^2}$

55. a. Use substitution to find the *numerical* value of $\frac{x^2 - 5x}{x - 5}$, then reduce each *numerical fraction* to lowest terms when:

(1) $x = 7$ (2) $x = 10$ (3) $x = 20$

(4) $x = 2$ (5) $x = -4$ (6) $x = -10$

b. What pattern, if any, do you observe for the answers to part **a**?

c. Can substitution be used to evaluate $\frac{x^2 - 5x}{x - 5}$ when $x = 5$? Explain your answer.

d. Reduce the algebraic fraction $\frac{x^2 - 5x}{x - 5}$ to lowest terms.

e. Using the answer to part **d**, find the value of $\frac{x^2 - 5x}{x - 5}$, reduced to lowest terms, when $x = 38{,}756$.

f. If the fraction $\frac{x^2 - 5x}{x - 5}$ is multiplied by $\frac{x}{x}$ to become $\frac{x(x^2 - 5x)}{x(x - 5)}$, will it be equivalent to $\frac{x^2 - 5x}{x - 5}$? Explain your answer.

14-3 MULTIPLYING FRACTIONS

The product of two fractions is a fraction with the following properties:

1. The numerator is the product of the numerators of the given fractions.

2. The denominator is the product of the denominators of the given fractions.

In general, for any numbers a, b, x, and y, when $b \neq 0$ and $y \neq 0$:

$$\frac{a}{b} \cdot \frac{x}{y} = \frac{ax}{by}$$

We can find the product of $\frac{7}{27}$ and $\frac{9}{4}$ in lowest terms by using either of two methods.

METHOD 1. $\frac{7}{27} \times \frac{9}{4} = \frac{7 \times 9}{27 \times 4} = \frac{63}{108} = \frac{7 \times 9}{12 \times 9} = \frac{7}{12} \times \frac{9}{9} = \frac{7}{12} \times 1 = \frac{7}{12}$

METHOD 2. $\frac{7}{27} \times \frac{9}{4} = \frac{7}{\underset{3}{\cancel{27}}} \times \frac{\overset{1}{\cancel{9}}}{4} = \frac{7}{12}$

Notice that Method 2 requires less computation than Method 1 since the reduced form of the product was obtained by dividing the numerator and the denominator by a common factor *before* the product was found. This method may be called the **cancellation method**.

The properties that apply to the multiplication of arithmetic fractions also apply to the multiplication of algebraic fractions.

To multiply $\frac{5x^2}{7y}$ by $\frac{14y^2}{15x^3}$ and express the product in lowest terms, we may use either of the two methods. In this example, $x \neq 0$ and $y \neq 0$.

METHOD 1. $\frac{5x^2}{7y} \cdot \frac{14y^2}{15x^3} = \frac{5x^2 \cdot 14y^2}{7y \cdot 15x^3} = \frac{70x^2y^2}{105x^3y} = \frac{2y}{3x} \cdot \frac{35x^2y}{35x^2y} = \frac{2y}{3x} \cdot 1 = \frac{2y}{3x}$

METHOD 2. $\frac{5x^2}{7x} \cdot \frac{14y^2}{15x^3} = \frac{\overset{1}{\cancel{5x^2}}}{\underset{1}{\cancel{7y}}} \cdot \frac{\overset{2y}{\cancel{14y^2}}}{\underset{3x}{\cancel{15x^3}}} = \frac{2y}{3x}$ (the cancellation method)

While Method 1 is longer, it has the advantage of displaying each step as a property of fractions. This can be helpful for checking work.

Procedure

To multiply fractions:

METHOD 1

1. Multiply the numerators of the given fractions.
2. Multiply the denominators of the given fractions.
3. Reduce the resulting fraction, if possible, to lowest terms.

METHOD 2

1. Factor any polynomial that is not a monomial.
2. Use cancellation to divide a numerator and a denominator by each common factor.
3. Multiply the resulting numerators and the resulting denominators to write the product in lowest terms.

EXAMPLE 1

Multiply and express the product in reduced form: $\frac{5a^3}{9bx} \cdot \frac{6bx}{a^2}$

Solution **METHOD 1**

(1) Multiply the numerators and denominators of the given fractions:

$$\frac{5a^3}{9bx} \cdot \frac{6bx}{a^2} = \frac{5a^3 \cdot 6bx}{9bx \cdot a^2} = \frac{30a^3bx}{9a^2bx}$$

(2) Reduce the resulting fraction to lowest terms:

$$= \frac{10a}{3} \cdot \frac{3a^2bx}{3a^2bx} = \frac{10a}{3} \cdot 1 = \frac{10a}{3}$$

METHOD 2

(1) Divide the numerators and denominators by the common factors $3bx$ and a^2:

$$\frac{5a^3}{9bx} \cdot \frac{6bx}{a^2} = \frac{\cancel{5a^3}^{\,a}}{\cancel{9bx}_{\,3}} \cdot \frac{\cancel{6bx}^{\,2}}{\cancel{a^2}_{\,1}}$$

(2) Multiply the resulting numerators and the resulting denominators:

$$= \frac{10a}{3}$$

Answer $\frac{10a}{3}$ $(a \neq 0, b \neq 0, x \neq 0)$

EXAMPLE 2

Multiply and express the product in simplest form: $12a \cdot \frac{3}{8a}$

Solution Think of $12a$ as $\frac{12a}{1}$.

$$\begin{aligned} 12a \cdot \frac{3}{8a} &= \frac{12a}{1} \cdot \frac{3}{8a} = \frac{36a}{8a} \\ &= \frac{4a}{4a} \cdot \frac{9}{2} = 1 \cdot \frac{9}{2} = \frac{9}{2} \end{aligned}$$

Answer $\frac{9}{2}$ $(a \neq 0)$

EXAMPLE 3

Multiply and simplify the product: $\frac{x^2 - 5x + 6}{3x} \cdot \frac{2}{4x - 12}$

Solution $\frac{x^2 - 5x + 6}{3x} \cdot \frac{2}{4x - 12} = \frac{\cancel{(x - 3)}^{\,1}(x - 2)}{3x} \cdot \frac{\cancel{2}^{\,1}}{\cancel{4}_{\,2}\cancel{(x - 3)}_{\,1}} = \frac{x - 2}{6x}$

Answer $\frac{x - 2}{6x}$ $(x \neq 0, 3)$

EXERCISES

Writing About Mathematics

1. When reduced to lowest terms, a fraction whose numerator is $x^2 - 3x + 2$ equals -1. What is the denominator of the fraction? Explain your answer.

2. Does $\frac{x^2}{xz + z^2} \cdot \frac{x^2 - z^2}{x^2 - xz} = \frac{x}{z}$ for all values of x and z? Explain your answer.

Developing Skills

In 3–41, find each product in lowest terms. In each case, list any values of the variable for which the fractions are not defined.

3. $\frac{8}{12} \cdot \frac{30a}{36}$ **4.** $36 \cdot \frac{5y}{9y}$ **5.** $\frac{1}{2} \cdot 20x$ **6.** $\frac{5}{d} \cdot d^2$ **7.** $\frac{x^2}{36} \cdot 20$

8. $mn \cdot \frac{8}{m^2n^2}$ **9.** $\frac{24x}{35y} \cdot \frac{14y}{8x}$ **10.** $\frac{12x}{5y} \cdot \frac{15y^2}{36x^2}$ **11.** $\frac{m^2}{8} \cdot \frac{32}{3m}$ **12.** $\frac{6r^2}{5s^2} \cdot \frac{10rs}{6r^3}$

13. $\frac{30m^2}{18n} \cdot \frac{6n}{5m}$ **14.** $\frac{24a^3b^2}{7c^3} \cdot \frac{21c^2}{12ab}$ **15.** $\frac{7}{8} \cdot \frac{2x+4}{21}$ **16.** $\frac{3a+9}{15a} \cdot \frac{a^3}{18}$ **17.** $\frac{5x-5y}{x^2y} \cdot \frac{xy^2}{25}$

18. $\frac{12a-4}{b} \cdot \frac{b^3}{12}$ **19.** $\frac{ab-a}{b^2} \cdot \frac{b^3-b^2}{a}$ **20.** $\frac{x^2-1}{x^2} \cdot \frac{3x^2-3x}{15}$

21. $\frac{2r}{r-1} \cdot \frac{r-1}{10}$ **22.** $\frac{7s}{s+2} \cdot \frac{2s+4}{21}$ **23.** $\frac{8x}{2x+6} \cdot \frac{x+3}{x^2}$

24. $\frac{1}{x^2-1} \cdot \frac{2x+2}{6}$ **25.** $\frac{a^2-9}{3} \cdot \frac{12}{2a-6}$ **26.** $\frac{x^2-x-2}{3} \cdot \frac{21}{x^2-4}$

27. $\frac{a(a-b)^2}{4b} \cdot \frac{4b}{a(a^2-b^2)}$ **28.** $\frac{(a-2)^2}{4b} \cdot \frac{16b^3}{4-a^2}$ **29.** $\frac{a^2-7a-8}{2a+2} \cdot \frac{5}{a-8}$

30. $\frac{x^2+6x+5}{9y^2} \cdot \frac{3y}{x+1}$ **31.** $\frac{y^2-2y-3}{2c^3} \cdot \frac{4c^2}{2y+2}$ **32.** $\frac{4a-6}{4a+8} \cdot \frac{6a+12}{5a-15}$

33. $\frac{x^2-25}{4x^2-9} \cdot \frac{2x+3}{x-5}$ **34.** $\frac{4x+8}{6x+18} \cdot \frac{5x+15}{x^2-4}$ **35.** $\frac{y^2-81}{(y+9)^2} \cdot \frac{10y+90}{5y-45}$

36. $\frac{8x}{2x^2-8} \cdot \frac{8x+16}{32x^2}$ **37.** $\frac{2-x}{2x} \cdot \frac{3x}{3x-6}$ **38.** $\frac{x^2-3x+2}{2x^2-2} \cdot \frac{2x}{x-2}$

39. $\frac{b^2+81}{b^2-81} \cdot \frac{81-b^2}{81+b^2}$ **40.** $\frac{d^2-25}{4-d^2} \cdot \frac{5d^2-20}{d+5}$ **41.** $\frac{a^2+12a+36}{a^2-36} \cdot \frac{36-a^2}{36+a^2}$

42. What is the value of $\frac{x^2-4}{6x+12} \cdot \frac{4x-12}{x^2-5x+6}$ when $x = 65{,}908$?

14-4 DIVIDING FRACTIONS

We know that the operation of division may be defined in terms of the multiplicative inverse, the reciprocal. A quotient can be expressed as the dividend times the reciprocal of the divisor. Thus:

$$8 \div 5 = \frac{8}{1} \times \frac{1}{5} = \frac{8 \times 1}{1 \times 5} = \frac{8}{5} \qquad \text{and} \qquad \frac{8}{7} \div \frac{5}{3} = \frac{8}{7} \times \frac{3}{5} = \frac{8 \times 3}{7 \times 5} = \frac{24}{35}$$

We use the same rule to divide algebraic fractions. In general, for any numbers a, b, c, and d, when $b \neq 0$, $c \neq 0$, and $d \neq 0$:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Procedure

To divide by a fraction, multiply the dividend by the reciprocal of the divisor.

EXAMPLE 1

Divide: $\frac{16c^3}{21d^2} \div \frac{24c^4}{14d^3}$

Solution *How to Proceed*

(1) Multiply the dividend by the reciprocal of the divisor:

$$\frac{16c^3}{21d^2} \div \frac{24c^4}{14d^3} = \frac{16c^3}{21d^2} \cdot \frac{14d^3}{24c^4}$$

(2) Divide the numerators and denominators by the common factors:

$$= \frac{\overset{2}{\cancel{16c^3}}}{\underset{3}{\cancel{21d^2}}} \cdot \frac{\overset{2d}{\cancel{14d^3}}}{\underset{3c}{\cancel{24c^4}}}$$

(3) Multiply the resulting numerators and the resulting denominators:

$$= \frac{4d}{9c}$$

Answer $\frac{4d}{9c}$ $(c \neq 0, d \neq 0)$

EXAMPLE 2

Divide: $\frac{8x + 24}{x^2 - 25} \div \frac{4x}{x^2 + 8x + 15}$

Solution *How to Proceed*

(1) Multiply the dividend by the reciprocal of the divisor:

$$\frac{8x + 24}{x^2 - 25} \div \frac{4x}{x^2 + 8x + 15} = \frac{8x + 24}{x^2 - 25} \cdot \frac{x^2 + 8x + 15}{4x}$$

(2) Factor the numerators and denominators, and divide by the common factors:

$$= \frac{\overset{2}{\cancel{8}}(x + 3)}{\underset{1}{\cancel{(x + 5)}}(x - 5)} \cdot \frac{\overset{1}{\cancel{(x + 5)}}(x + 3)}{\underset{1}{\cancel{4}}x}$$

(3) Multiply the resulting numerators and the resulting denominators:

$$= \frac{2(x + 3)^2}{x(x - 5)}$$

Answer $\frac{2(x + 3)^2}{x(x - 5)}$ $(x \neq 0, 5, -5, -3)$

Note: If $x = 5, -5$, or -3, the dividend and the divisor will not be defined. If $x = 0$, the reciprocal of the divisor will not be defined.

EXERCISES

Writing About Mathematics

1. Explain why the quotient $\frac{2}{x-2} \div \frac{x-3}{5}$ is undefined for $x = 2$ and for $x = 3$.

2. To find the quotient $\frac{3}{2(x-4)} \div \frac{x-4}{5}$, Ruth canceled $(x - 4)$ in the numerator and denominator and wrote $\frac{3}{2} \div \frac{1}{5} = \frac{3}{2} \cdot \frac{5}{1} = \frac{15}{2}$. Is Ruth's answer correct? Explain why or why not.

Developing Skills

In 3–27, find each quotient in lowest terms. In each case, list any values of the variables for which the quotient is not defined.

3. $\frac{7a}{10} \div \frac{21}{5}$ **4.** $\frac{12}{35} \div \frac{4b}{7}$ **5.** $8 \div \frac{x}{2y}$ **6.** $\frac{x}{9} \div \frac{x}{3}$

7. $\frac{3x}{5y} \div \frac{21x}{2y}$ **8.** $\frac{7ab^2}{10cd} \div \frac{14b^3}{5c^2d^2}$ **9.** $\frac{xy^2}{x^2y} \div \frac{x}{y^3}$ **10.** $\frac{6a^2b^2}{8c} \div 3ab$

11. $\frac{4x+4}{9} \div \frac{3}{8x}$ **12.** $\frac{3y^2+9y}{18} \div \frac{5y^2}{27}$ **13.** $\frac{a^3-a}{b} \div \frac{a^3}{4b^3}$ **14.** $\frac{x^2-1}{5} \div \frac{x-1}{10}$

15. $\frac{x^2-5x+4}{2x} \div \frac{2x-2}{8x^2}$ **16.** $\frac{4a^2-9}{10} \div \frac{10a+15}{25}$ **17.** $\frac{b^2-b-6}{2b} \div \frac{b^2-4}{b^2}$

18. $\frac{a^2-ab}{4a} \div (a^2-b^2)$ **19.** $\frac{12y-6}{8} \div (2y^2-3y+1)$ **20.** $\frac{(x-2)^2}{4x^2-16} \div \frac{21x}{3x+6}$

21. $\frac{x^2-2xy-8y^2}{x^2-16y^2} \div \frac{5x+10y}{3x+12y}$ **22.** $\frac{x^2-4x+4}{3x-6} \div (2-x)$ **23.** $(9-y^2) \div \frac{y^2+8y+15}{2y+10}$

24. $\frac{x-1}{x+1} \cdot \frac{2x+2}{x+2} \div \frac{4x-4}{x+2}$ **25.** $\frac{x+y}{x^2+y^2} \cdot \frac{x}{x-y} \div \frac{(x+y)^2}{x^4-y^4}$ **26.** $\frac{2a+6}{a^2-9} \div \frac{3+a}{3-a} \cdot \frac{a+3}{4}$

27. $\frac{(a+b)^2}{a^2-b^2} \div \frac{a+b}{b^2-a^2} \cdot \frac{a-b}{(a-b)^2}$

28. For what value(s) of a is $\frac{a^2-2a+1}{a^2} \div \frac{a^2-1}{a}$ undefined?

29. Find the value of $\frac{y^2-6y+9}{y^2-9} \div \frac{10y-30}{y^2+3y}$ when $y = 70$.

30. If $x \div y = a$ and $y \div z = \frac{1}{a}$, what is the value of $x \div z$?

14-5 ADDING OR SUBTRACTING ALGEBRAIC FRACTIONS

We know that the sum (or difference) of two arithmetic fractions that have the same denominator is another fraction whose numerator is the sum (or difference) of the numerators and whose denominator is the common denominator of the given fractions. We use the same rule to add algebraic fractions that have the same nonzero denominator. Thus:

Arithmetic fractions	*Algebraic fractions*
$\frac{5}{7} + \frac{1}{7} = \frac{5+1}{7} = \frac{6}{7}$	$\frac{a}{x} + \frac{b}{x} = \frac{a+b}{x}$
$\frac{5}{7} - \frac{1}{7} = \frac{5-1}{7} = \frac{4}{7}$	$\frac{a}{x} - \frac{b}{x} = \frac{a-b}{x}$

Procedure

To add (or subtract) fractions that have the same denominator:

1. Write a fraction whose numerator is the sum (or difference) of the numerators and whose denominator is the common denominator of the given fractions.
2. Reduce the resulting fraction to lowest terms.

EXAMPLE 1

Add and reduce the answer to lowest terms: $\frac{5}{4x} + \frac{9}{4x}$

Solution $\frac{5}{4x} + \frac{9}{4x} = \frac{5+9}{4x} = \frac{14}{4x} = \frac{7}{2x}$

Answer $\frac{7}{2x}$ $(x \neq 0)$

EXAMPLE 2

Subtract: $\frac{4x+7}{6x} - \frac{2x-4}{6x}$

Solution $\frac{4x+7}{6x} - \frac{2x-4}{6x} = \frac{(4x+7)-(2x-4)}{6x} = \frac{4x+7-2x+4}{6x} = \frac{2x+11}{6x}$

Answer $\frac{2x+11}{6x}$ $(x \neq 0)$

Note: In Example 2, since the fraction bar is a symbol of grouping, we enclose numerators in parentheses when the difference is written as a single fraction. In this way, we can see all the signs that need to be changed for the subtraction.

In arithmetic, in order to add (or subtract) fractions that have *different* denominators, we change these fractions to equivalent fractions that have the *same* denominator, called the **common denominator**. Then we add (or subtract) the equivalent fractions.

For example, to add $\frac{3}{4}$ and $\frac{1}{6}$, we use any common denominator that has 4 and 6 as factors.

METHOD 1. Use the product of the denominators as the common denominator. Here, a common denominator is 4×6, or 24.

$$\frac{3}{4} + \frac{1}{6} = \frac{3}{4} \times \frac{6}{6} + \frac{1}{6} \times \frac{4}{4} = \frac{18}{24} + \frac{4}{24} = \frac{22}{24} = \frac{11}{12}$$ *Answer*

METHOD 2. To simplify our work, we use the **least common denominator (LCD)**, that is, the least common multiple of the given denominators. The LCD of $\frac{3}{4}$ and $\frac{1}{6}$ is 12.

$$\frac{3}{4} + \frac{1}{6} = \frac{3}{4} \times \frac{3}{3} + \frac{1}{6} \times \frac{2}{2} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12} \quad \textit{Answer}$$

To find the least common denominator of two fractions, we factor the denominators of the fractions completely. The LCD is the product of all of the factors of the first denominator times the factors of the second denominator that are not factors of the first.

$$\begin{aligned} 4 &= 2 \cdot 2 \\ 6 &= 2 \cdot 3 \\ \text{LCD} &= 2 \cdot 2 \cdot 3 \end{aligned}$$

Then, to change each fraction to an equivalent form that has the LCD as the denominator, we multiply by $\frac{x}{x}$, where x is the number by which the original denominator must be multiplied to obtain the LCD.

$$\frac{3}{4}\left(\frac{x}{x}\right) = \frac{}{12} \qquad \frac{1}{6}\left(\frac{x}{x}\right) = \frac{}{12}$$

$$\frac{3}{4}\left(\frac{3}{3}\right) = \frac{9}{12} \qquad \frac{1}{6}\left(\frac{2}{2}\right) = \frac{2}{12}$$

Note that the LCD is the smallest possible common denominator.

Procedure

To add (or subtract) fractions that have different denominators:

1. Choose a common denominator for the fractions.
2. Change each fraction to an equivalent fraction with the chosen common denominator.
3. Write a fraction whose numerator is the sum (or difference) of the numerators of the new fractions and whose denominator is the common denominator.
4. Reduce the resulting fraction to lowest terms.

Algebraic fractions are added in the same manner as arithmetic fractions, as shown in the examples that follow.

EXAMPLE 3

Add: $\frac{5}{a^2b} + \frac{2}{ab^2}$

Solution *How to Proceed*

(1) Find the LCD of the fractions:

$$\begin{aligned} a^2b &= a \cdot a \cdot b \\ ab^2 &= a \cdot b \cdot b \\ \text{LCD} &= a \cdot a \cdot b \cdot b = a^2b^2 \end{aligned}$$

(2) Change each fraction to an equivalent fraction with the least common denominator, a^2b^2:

$$\frac{5}{a^2b} + \frac{2}{ab^2} = \frac{5}{a^2b} \cdot \frac{b}{b} + \frac{2}{ab^2} \cdot \frac{a}{a} = \frac{5b}{a^2b^2} + \frac{2a}{a^2b^2}$$

(3) Write a fraction whose numerator is the sum of the numerators of the new fractions and whose denominator is the common denominator:

$$= \frac{5b + 2a}{a^2b^2}$$

Answer $\frac{5b + 2a}{a^2b^2}$ $(a \neq 0, b \neq 0)$

EXAMPLE 4

Subtract: $\frac{2x + 5}{3} - \frac{x - 2}{4}$

Solution

$$\text{LCD} = 3 \cdot 4 = 12$$

$$\begin{aligned} \frac{2x + 5}{3} - \frac{x - 2}{4} &= \frac{4}{4} \cdot \frac{2x + 5}{3} - \frac{3}{3} \cdot \frac{x - 2}{4} \\ &= \frac{8x + 20}{12} - \frac{3x - 6}{12} \\ &= \frac{(8x + 20) - (3x - 6)}{12} \\ &= \frac{8x + 20 - 3x + 6}{12} \\ &= \frac{5x + 26}{12} \quad \textit{Answer} \end{aligned}$$

EXAMPLE 5

Express as a fraction in simplest form: $y + 1 - \frac{1}{y - 1}$

Solution LCD $= y - 1$

$$y + 1 - \frac{1}{y - 1} = \frac{y + 1}{1} \cdot \left(\frac{y - 1}{y - 1}\right) - \frac{1}{y - 1} = \frac{y^2 - 1}{y - 1} - \frac{1}{y - 1} = \frac{y^2 - 1 - 1}{y - 1} = \frac{y^2 - 2}{y - 1}$$

Answer $\frac{y^2 - 2}{y - 1}$ $(y \neq 1)$

EXAMPLE 6

Subtract: $\frac{6x}{x^2 - 4} - \frac{3}{x - 2}$

Solution

$$x^2 - 4 = (x - 2)(x + 2)$$
$$x - 2 = (x - 2)$$
$$\text{LCD} = (x - 2)(x + 2)$$

$$\begin{aligned}\frac{6x}{x^2 - 4} - \frac{3}{x - 2} &= \frac{6x}{(x - 2)(x + 2)} - \frac{3(x + 2)}{(x - 2)(x + 2)}\\ &= \frac{6x - 3(x + 2)}{(x - 2)(x + 2)}\\ &= \frac{6x - 3x - 6}{(x - 2)(x + 2)}\\ &= \frac{3x - 6}{(x - 2)(x + 2)}\\ &= \frac{3(x - 2)}{(x - 2)(x + 2)}\\ &= \frac{3}{x + 2}\end{aligned}$$

Answer $\frac{3}{x + 2}$ $(x \neq 2, -2)$

EXERCISES

Writing About Mathematics

1. In Example 2, the answer is $\frac{2x + 11}{6x}$. Can we divide $2x$ and $6x$ by 2 to write the answer in lowest terms as $\frac{x + 11}{3x}$? Explain why or why not.

2. Joey said that $2 - \frac{x - a}{a - x} = 3$ if $a \neq x$. Do you agree with Joey? Explain why or why not.

Developing Skills

In 3–43, add or subtract the fractions as indicated. Reduce each answer to lowest terms. In each case, list the values of the variables for which the fractions are not defined.

3. $\frac{11}{4c} + \frac{5}{4c} - \frac{6}{4c}$

4. $\frac{5r}{t} - \frac{2s}{t}$

5. $\frac{6}{10c} + \frac{9}{10c} - \frac{3}{10c}$

6. $\frac{x}{x + 1} + \frac{1}{x + 1}$

7. $\frac{6y - 4}{4y + 3} + \frac{7 - 2y}{4y + 3}$

8. $\frac{9d + 6}{2d + 1} - \frac{7d + 5}{2d + 1}$

9. $\frac{6x - 5}{x^2 - 1} - \frac{5x - 6}{x^2 - 1}$

10. $\frac{r^2 + 4r}{r^2 - r - 6} + \frac{8 - r^2}{r^2 - r - 6}$

11. $\frac{x}{3} + \frac{x}{2}$

12. $\frac{5x}{6} - \frac{2x}{3}$

13. $\frac{y}{6} + \frac{y}{5} - \frac{y}{2}$

14. $\frac{ab}{5} + \frac{ab}{4}$

15. $\frac{8x}{5} - \frac{3x}{4} + \frac{7x}{10}$

16. $\frac{5a}{6} - \frac{3a}{4}$

17. $\frac{a}{7} + \frac{b}{14}$

18. $\frac{9}{4x} + \frac{3}{2x}$

19. $\frac{1}{2x} - \frac{1}{x} + \frac{1}{8x}$

20. $\frac{9a}{8b} - \frac{3a}{4b}$

21. $d + \frac{7}{5d}$

22. $\frac{a-3}{3} + \frac{a+1}{6}$

23. $\frac{3y-4}{5} - \frac{y-2}{4}$

24. $\frac{b-3}{5b} - \frac{b+2}{10b}$

25. $\frac{y-4}{4y^2} + \frac{3y-5}{3y}$

26. $\frac{3c-7}{2c} - \frac{3c-3}{6c^2}$

27. $3 + \frac{5}{x+1}$

28. $5 - \frac{2x}{x+y}$

29. $\frac{5}{x-3} + \frac{7}{2x-6}$

30. $\frac{9}{y+1} - \frac{3}{4y+4}$

31. $\frac{2}{3a-1} + \frac{7}{15a-5}$

32. $\frac{10}{3x-6} + \frac{3}{2x-4}$

33. $\frac{1x}{8x-8} - \frac{3x}{4x-4}$

34. $\frac{5}{y^2-9} - \frac{3}{y-3}$

35. $\frac{6}{y^2-16} - \frac{5}{y+4}$

36. $\frac{x}{x^2-36} - \frac{4}{3x+18}$

37. $\frac{1}{y-3} + \frac{2}{y+4} + \frac{2}{3}$

38. $a + 1 + \frac{1}{a+1}$

39. $x - 5 - \frac{x}{x+3}$

40. $\frac{2x-1}{x+2} + 2x - 3$

41. $\frac{x+2y}{3x+12y} - \frac{6x-y}{x^2+3xy-4y^2}$

42. $\frac{7a}{(a-1)(a+3)} + \frac{2a-5}{(a+3)(a+2)}$

43. $\frac{2a+7}{a^2-2a-15} - \frac{3a-4}{a^2-7a+10}$

Applying Skills

In 44–46, represent the perimeter of each polygon in simplest form.

44. The lengths of the sides of a triangle are represented by $\frac{x}{2}$, $\frac{3x}{5}$, and $\frac{7x}{10}$.

45. The length of a rectangle is represented by $\frac{x+3}{4}$, and its width is represented by $\frac{x-4}{3}$.

46. Each leg of an isosceles triangle is represented by $\frac{2x-3}{7}$, and its base is represented by $\frac{6x-18}{21}$.

In 47 and 48, find, in each case, the simplest form of the indicated length.

47. The perimeter of a triangle is $\frac{17x}{24}$, and the lengths of two of the sides are $\frac{3x}{8}$ and $\frac{2x-5}{12}$. Find the length of the third side.

48. The perimeter of a rectangle is $\frac{14x}{15}$, and the measure of each length is $\frac{x+2}{3}$. Find the measure of each width.

49. The time t needed to travel a distance d at a rate of speed r can be found by using the formula $t = \frac{d}{r}$.

a. For the first part of a trip, a car travels x miles at 45 miles per hour. Represent the time that the car traveled at that speed in terms of x.

b. For the remainder of the trip, the car travels $2x + 20$ miles at 60 miles per hour. Represent the time that the car traveled at that speed in terms of x.

c. Express, in terms of x, the total time for the two parts of the trip.

50. Ernesto walked 2 miles at a miles per hour and then walked 3 miles at $(a - 1)$ miles per hour. Represent, in terms of a, the total time that he walked.

51. Fran rode her bicycle for x miles at 10 miles per hour and then rode $(x + 3)$ miles farther at 8 miles per hour. Represent, in terms of x, the total time that she rode.

14-6 SOLVING EQUATIONS WITH FRACTIONAL COEFFICIENTS

The following equations contain fractional coefficients:

$$\tfrac{1}{2}x = 10 \qquad \tfrac{x}{2} = 10 \qquad \tfrac{1}{3}x + 60 = \tfrac{5}{6}x \qquad \tfrac{x}{3} + 60 = \tfrac{5x}{6}$$

Each of these equations can be solved by finding an equivalent equation that does not contain fractional coefficients. This can be done by multiplying both sides of the equation by a common denominator for all the fractions present in the equation. We usually multiply by the least common denominator, the LCD.

Note that the equation $0.5x = 10$ can also be written as $\tfrac{1}{2}x = 10$, since a decimal fraction can be replaced by a common fraction.

Procedure

To solve an equation that contains fractional coefficients:

1. Find the LCD of all coefficients.
2. Multiply both sides of the equation by the LCD.
3. Solve the resulting equation using the usual methods.
4. Check in the original equation.

EXAMPLE 1

Solve and check: $\tfrac{x}{3} + \tfrac{x}{5} = 8$

Solution *How to Proceed*

(1) Write the equation: $\tfrac{x}{3} + \tfrac{x}{5} = 8$

(2) Find the LCD: $\text{LCD} = 3 \cdot 5 = 15$

(3) Multiply both sides of the equation by the LCD: $15\left(\tfrac{x}{3} + \tfrac{x}{5}\right) = 15(8)$

(4) Use the distributive property: $15\left(\tfrac{x}{3}\right) + 15\left(\tfrac{x}{5}\right) = 15(8)$

(5) Simplify: $5x + 3x = 120$

(6) Solve for x: $8x = 120$

$x = 15$

Check

$$\tfrac{x}{3} + \tfrac{x}{5} = 8$$

$$\tfrac{\mathbf{15}}{3} + \tfrac{\mathbf{15}}{5} \stackrel{?}{=} 8$$

$$5 + 3 \stackrel{?}{=} 8$$

$$8 = 8 \checkmark$$

Answer $x = 15$

EXAMPLE 2

Solve:

a. $\frac{3x}{4} = 20 + \frac{x}{4}$ **b.** $\frac{2x+7}{6} - \frac{2x-9}{10} = 3$

Solution

a.

$$\begin{aligned} \frac{3x}{4} &= 20 + \frac{x}{4} \\ \text{LCD} &= 4 \\ 4\left(\frac{3x}{4}\right) &= 4\left(20 + \frac{x}{4}\right) \\ 4\left(\frac{3x}{4}\right) &= 4(20) + 4\left(\frac{x}{4}\right) \\ 3x &= 80 + x \\ 2x &= 80 \\ x &= 40 \quad \textit{Answer} \end{aligned}$$

b.

$$\begin{aligned} \frac{2x+7}{6} - \frac{2x-9}{10} &= 3 \\ \text{LCD} &= 30 \\ 30\left(\frac{2x+7}{6} - \frac{2x-9}{10}\right) &= 30(3) \\ 30\left(\frac{2x+7}{6}\right) - 30\left(\frac{2x-9}{10}\right) &= 30(3) \\ 5(2x+7) - 3(2x-9) &= 90 \\ 10x + 35 - 6x + 27 &= 90 \\ 4x + 62 &= 90 \\ 4x &= 28 \\ x &= 7 \quad \textit{Answer} \end{aligned}$$

In Example 2, the check is left to you.

EXAMPLE 3

A woman purchased stock in the PAX Company over 3 months. In the first month, she purchased one-half of her present number of shares. In the second month, she bought two-fifths of her present number of shares. In the third month, she purchased 14 shares. How many shares of PAX stock did the woman purchase?

Solution Let x = total number of shares of stock purchased.

Then $\frac{1}{2}x$ = number of shares purchased in month 1,

$\frac{2}{5}x$ = number of shares purchased in month 2,

14 = number of shares purchased in month 3.

The sum of the shares purchased over 3 months is the total number of shares.

month 1 + month 2 + month 3 = total

$$\begin{aligned} \tfrac{1}{2}x + \tfrac{2}{5}x + 14 &= x \\ 10\left(\tfrac{1}{2}x + \tfrac{2}{5}x + 14\right) &= 10(x) \\ 5x + 4x + 140 &= 10x \\ 9x + 140 &= 10x \\ 140 &= x \end{aligned}$$

Check month 1 $= \frac{1}{2}(140) = 70$, month 2 $= \frac{2}{5}(140) = 56$, month 3 $= 14$

$$70 + 56 + 14 = 140 \checkmark$$

Answer 140 shares

EXAMPLE 4

In a child's coin bank, there is a collection of nickels, dimes, and quarters that amounts to \$3.20. There are 3 times as many quarters as nickels, and 5 more dimes than nickels. How many coins of each kind are there?

Solution

$$\begin{aligned} \text{Let } x &= \text{the number of nickels.} \\ \text{Then } 3x &= \text{the number of quarters,} \\ \text{and } x + 5 &= \text{the number of dimes.} \\ \text{Also, } 0.05x &= \text{the value of the nickels,} \\ 0.25(3x) &= \text{the value of the quarters,} \\ \text{and } 0.10(x + 5) &= \text{the value of the dimes.} \end{aligned}$$

Write the equation for the value of the coins. To simplify the equation, which contains coefficients that are decimal fractions with denominators of 100, multiply each side of the equation by 100.

The total value of the coins is \$3.20.

$$\begin{aligned} 0.05x + 0.25(3x) + 0.10(x + 5) &= 3.20 \\ 100[0.05x + 0.25(3x) + 0.10(x + 5)] &= 100(3.20) \\ 5x + 25(3x) + 10(x + 5) &= 320 \\ 5x + 75x + 10x + 50 &= 320 \\ 90x + 50 &= 320 \\ 90x &= 270 \\ x &= 3 \end{aligned}$$

Check There are 3 nickels, $3(3) = 9$ quarters, and $3 + 5 = 8$ dimes.

$$\begin{aligned} &\text{The value of 3 nickels is } \$0.05(3) &&= \$0.15 \\ &\text{The value of 9 quarters is } \$0.25(9) &&= \$2.25 \\ &\text{The value of 8 dimes is } \$0.10(8) &&= \underline{\$0.80} \\ & && \quad \$3.20 \checkmark \end{aligned}$$

Answer There are 3 nickels, 9 quarters, and 8 dimes.

Note: In a problem such as this, a chart such as the one shown below can be used to organize the information:

Coins	Number of Coins	Value of One Coin	Total Value
Nickels	x	0.05	$0.05x$
Quarters	$3x$	0.25	$0.25(3x)$
Dimes	$x + 5$	0.10	$0.10(x + 5)$

EXERCISES

Writing About Mathematics

1. Abby solved the equation $0.2x - 0.84 = 3x$ as follows:

$$\begin{array}{rcr} 0.2x - 0.84 & = & 3x \\ \underline{-0.2x \qquad\quad} & & \underline{-0.2x} \\ -0.84 & = & 0.1x \\ \\ -8.4 & = & x \end{array}$$

Is Abby's solution correct? Explain why or why not.

2. In order to write the equation $0.2x - 0.84 = 3x$ as an equivalent equation with integral coefficients, Heidi multiplied both sides of the equation by 10. Will Heidi's method lead to a correct solution? Explain why or why not. Compare Heidi's method with multiplying by 100 or multiplying by 1,000.

Developing Skills

In 3–37, solve each equation and check.

3. $\frac{x}{7} = 3$

4. $\frac{1}{6}t = 18$

5. $\frac{3x}{5} = 15$

6. $\frac{x + 8}{4} = 6$

7. $\frac{m - 2}{9} = 3$

8. $\frac{2r + 6}{5} = -4$

9. $\frac{5y - 30}{7} = 0$

10. $\frac{5x}{2} = \frac{15}{4}$

11. $\frac{m - 5}{35} = \frac{5}{7}$

12. $\frac{2x + 1}{3} = \frac{6x - 9}{5}$

13. $\frac{3y + 1}{4} = \frac{44 - y}{5}$

14. $\frac{x}{5} + \frac{x}{3} = \frac{8}{15}$

15. $10 = \frac{x}{3} + \frac{x}{7}$

16. $\frac{r}{3} - \frac{r}{6} = 2$

17. $\frac{3t}{4} - 6 = \frac{t}{12}$

18. $\frac{a}{2} + \frac{a}{3} + \frac{a}{4} = 26$

19. $\frac{7y}{12} - \frac{1}{4} = 2y - \frac{5}{3}$

20. $\frac{y + 2}{4} - \frac{y - 3}{3} = \frac{1}{2}$

21. $\frac{t - 3}{6} - \frac{t - 25}{5} = 4$

22. $\frac{3m + 1}{4} = 2 - \frac{3 - 2m}{6}$

23. $0.03y - 1.2 = 8.7$

24. $0.4x + 0.08 = 4.24$

25. $2c + 0.5c = 50$

26. $0.08y - 0.9 = 0.02y$

27. $1.7x = 30 + 0.2x$

28. $0.02(x + 5) = 8$

29. $0.05(x - 8) = 0.07x$

30. $0.4(x - 9) = 0.3(x + 4)$

31. $0.06(x - 5) = 0.04(x + 8)$

32. $0.04x + 0.03(2{,}000 - x) = 75$

33. $0.02x + 0.04(1{,}500 - x) = 48$

34. $0.05x + 10 = 0.06(x + 50)$

35. $0.08x = 0.03(x + 200) - 4$

36. $\frac{0.4a}{3} + \frac{0.2a}{4} = 2$

37. $\frac{0.1a}{6} - \frac{0.3a}{4} = 3$

38. The sum of one-half of a number and one-third of that number is 25. Find the number.

39. The difference between one-fifth of a positive number and one-tenth of that number is 10. Find the number.

40. If one-half of a number is increased by 20, the result is 35. Find the number.

41. If two-thirds of a number is decreased by 30, the result is 10. Find the number.

42. If the sum of two consecutive integers is divided by 3, the quotient is 9. Find the integers.

43. If the sum of two consecutive odd integers is divided by 4, the quotient is 10. Find the integers.

44. In an isosceles triangle, each of the congruent sides is two-thirds of the base. The perimeter of the triangle is 42. Find the length of each side of the triangle.

45. The larger of two numbers is 12 less than 5 times the smaller. If the smaller number is equal to one-third of the larger number, find the numbers.

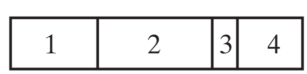
46. The larger of two numbers exceeds the smaller by 14. If the smaller number is equal to three-fifths of the larger, find the numbers.

47. Separate 90 into two parts such that one part is one-half of the other part.

48. Separate 150 into two parts such that one part is two-thirds of the other part.

Applying Skills

49. Four vegetable plots of unequal lengths and of equal widths are arranged as shown. The length of the third plot is one-fourth the length of the second plot.



The length of the fourth plot is one-half the length of the second plot. The length of the first plot is 10 feet more than the length of the fourth plot. If the total length of the four plots is 100 feet, find the length of each plot.

50. Sam is now one-sixth as old as his father. In 4 years, Sam will be one-fourth as old as his father will be then. Find the ages of Sam and his father now.

51. Robert is one-half as old as his father. Twelve years ago, he was one-third as old as his father was then. Find their present ages.

52. A coach finds that, of the students who try out for track, 65% qualify for the team and 90% of those who qualify remain on the team throughout the season. What is the smallest number of students who must try out for track in order to have 30 on the team at the end of the season?

53. A bus that runs once daily between the villages of Alpaca and Down makes only two stops in between, at Billow and at Comfort. Today, the bus left Alpaca with some passengers. At Billow, one-half of the passengers got off, and six new ones got on. At Comfort, again one-half of the passengers got off, and, this time, five new ones got on. At Down, the last 13 passengers on the bus got off. How many passengers were aboard when the bus left Alpaca?

54. Sally spent half of her money on a present for her mother. Then she spent one-quarter of the cost of the present for her mother on a treat for herself. If Sally had $6.00 left after she bought her treat, how much money did she have originally?

55. Bob planted some lettuce seedlings in his garden. After a few days, one-tenth of these seedlings had been eaten by rabbits. A week later, one-fifth of the remaining seedlings had been eaten, leaving 36 seedlings unharmed. How many lettuce seedlings had Bob planted originally?

56. May has 3 times as many dimes as nickels. In all, she has $1.40. How many coins of each type does she have?

57. Mr. Jantzen bought some cans of soup at $0.39 per can, and some packages of frozen vegetables at $0.59 per package. He bought twice as many packages of vegetables as cans of soup. If the total bill was $9.42, how many cans of soup did he buy?

58. Roger has $2.30 in dimes and nickels. There are 5 more dimes than nickels. Find the number of each kind of coin that he has.

59. Bess has $2.80 in quarters and dimes. The number of dimes is 7 less than the number of quarters. Find the number of each kind of coin that she has.

60. A movie theater sold student tickets for $5.00 and full-price tickets for $7.00. On Saturday, the theater sold 16 more full-price tickets than student tickets. If the total sales on Saturday were $1,072, how many of each kind of ticket were sold?

61. Is it possible to have $4.50 in dimes and quarters, and have twice as many quarters as dimes? Explain.

62. Is it possible to have $6.00 in nickels, dimes, and quarters, and have the same number of each kind of coin? Explain.

63. Mr. Symms invested a sum of money in 7% bonds. He invested $400 more than this sum in 8% bonds. If the total annual interest from these two investments is $257, how much did he invest at each rate?

64. Mr. Charles borrowed a sum of money at 10% interest. He borrowed a second sum, which was $1,500 less than the first sum, at 11% interest. If the annual interest on these two loans is $202.50, how much did he borrow at each rate?

14-7 SOLVING INEQUALITIES WITH FRACTIONAL COEFFICIENTS

In our modern world, many problems involve inequalities. A potential buyer may offer *at most* one amount for a house, while the seller will accept *no less* than another amount. Inequalities that contain fractional coefficients are handled in much the same way as equations that contain fractional coefficients. The chart on the right helps us to translate words into algebraic symbols.

Words	Symbols
a is greater than b	$a > b$
a is less than b	$a < b$
a is at least b a is no less than b	$a \geq b$
a is at most b a is no greater than b	$a \leq b$

Procedure

To solve an inequality that contains fractional coefficients:

1. Find the LCD, a positive number.
2. Multiply both sides of the inequality by the LCD.
3. Solve the resulting inequality using the usual methods.

EXAMPLE 1

Solve the inequality, and graph the solution set on a number line:

a. $\frac{x}{3} - \frac{x}{6} > 2$

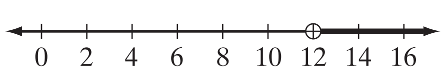
b. $\frac{3y}{2} + \frac{8 - 4y}{7} \leq 3$

Solution **a.**

$$\begin{aligned} \frac{x}{3} - \frac{x}{6} &> 2 \\ 6\left(\frac{x}{3} - \frac{x}{6}\right) &> 6(2) \\ 6\left(\frac{x}{3}\right) - 6\left(\frac{x}{6}\right) &> 12 \\ 2x - x &> 12 \\ x &> 12 \end{aligned}$$

Since no domain was given, use the domain of real numbers.

Answer: $x > 12$

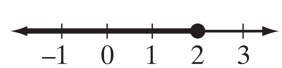


b.

$$\begin{aligned} \frac{3y}{2} + \frac{8 - 4y}{7} &\leq 3 \\ 14\left(\frac{3y}{2} + \frac{8 - 4y}{7}\right) &\leq 14(3) \\ 14\left(\frac{3y}{2}\right) + 14\left(\frac{8 - 4y}{7}\right) &\leq 42 \\ 21y + 16 - 8y &\leq 42 \\ 13y &\leq 26 \\ y &\leq 2 \end{aligned}$$

Since no domain was given, use the domain of real numbers.

Answer: $y \leq 2$



EXAMPLE 2

Two boys want to pool their money to buy a comic book. The younger of the boys has one-third as much money as the older. Together they have more than \$2.00. Find the smallest possible amount of money each can have.

Solution Let x = the number of cents that the older boy has.
Then $\frac{1}{3}x$ = the number of cents that the younger boy has.

The sum of their money in cents is greater than 200.

$$\begin{aligned} x + \tfrac{1}{3}x &> 200 \\ 3\left(x + \tfrac{1}{3}x\right) &> 3(200) \\ 3x + x &> 600 \\ 4x &> 600 \\ x &> 150 \\ \tfrac{1}{3}x &> 50 \end{aligned}$$

The number of cents that the younger boy has must be an integer greater than 50. The number of cents that the older boy has must be a multiple of 3 that is greater than 150. The younger boy has at least 51 cents. The older boy has at least 153 cents. The sum of 51 and 153 is greater than 200.

Answer The younger boy has at least \$0.51 and the older boy has at least \$1.53.

EXERCISES

Writing About Mathematics

1. Explain the error in the following solution of an inequality.

$$\begin{aligned} \tfrac{x}{-3} &> -2 \\ -3\left(\tfrac{x}{-3}\right) &> -3(-2) \\ x &> 6 \end{aligned}$$

2. In Example 2, what is the domain for the variable?

Developing Skills

In 3–23, solve each inequality, and graph the solution set on a number line.

3. $\frac{1}{4}x - \frac{1}{5}x > \frac{9}{20}$

4. $y - \frac{2}{3}y < 5$

5. $\frac{5}{6}c > \frac{1}{3}c + 3$

6. $\frac{x}{4} - \frac{x}{8} \leq \frac{5}{8}$

7. $\frac{y}{6} \geq \frac{y}{12} + 1$

8. $\frac{y}{9} - \frac{y}{4} > \frac{5}{36}$

9. $\frac{t}{10} \leq 4 + \frac{t}{5}$

10. $1 + \frac{2x}{3} \geq \frac{x}{2}$

11. $2.5x - 1.7x > 4$

12. $2y + 3 \geq 0.2y$

13. $\frac{3x - 1}{7} > 5$

14. $\frac{5y - 30}{7} \leq 0$

15. $2d + \frac{1}{4} < \frac{7d}{12} + \frac{5}{3}$

16. $\frac{4c}{3} - \frac{7}{9} \geq \frac{c}{2} + \frac{7}{6}$

17. $\frac{2m}{3} \geq \frac{7 - m}{4} + 1$

18. $\frac{3x - 30}{6} < \frac{x}{3} - 2$

19. $\frac{6x - 3}{2} > \frac{37}{10} + \frac{x + 2}{5}$

20. $\frac{2y - 3}{3} + \frac{y + 1}{2} < 10$

21. $\frac{2r - 3}{5} - \frac{r - 3}{3} \leq 2$

22. $\frac{3t - 4}{3} \geq \frac{2t + 4}{6} + \frac{5t - 1}{9}$

23. $\frac{2 - a}{2} \leq \frac{a + 2}{5} - \frac{2a + 3}{6}$

24. If one-third of an integer is increased by 7, the result is at most 13. Find the largest possible integer.

25. If two-fifths of an integer is decreased by 11, the result is at least 4. Find the smallest possible integer.

26. The sum of one-fifth of an integer and one-tenth of that integer is less than 40. Find the greatest possible integer.

27. The difference between three-fourths of a positive integer and one-half of that integer is greater than 28. Find the smallest possible integer.

28. The smaller of two integers is two-fifths of the larger, and their sum is less than 40. Find the largest possible integers.

29. The smaller of two positive integers is five-sixths of the larger, and their difference is greater than 3. Find the smallest possible integers.

Applying Skills

30. Talk and Tell Answering Service offers customers two monthly options.

OPTION 1 *Measured Service*
base rate is $15
each call costs $0.10

OPTION 2 *Unmeasured Service*
base rate is $20
no additional charge per call

Find the least number of calls for which unmeasured service is cheaper than measured service.

31. Paul earned some money mowing lawns. He spent one-half of this money for a book, and then one-third for a CD. If he had less than $3 left, how much money did he earn?

32. Mary bought some cans of vegetables at $0.89 per can, and some cans of soup at $0.99 per can. If she bought twice as many cans of vegetables as cans of soup, and paid at least $10, what is the least number of cans of vegetables she could have bought?

33. A coin bank contains nickels, dimes, and quarters. The number of dimes is 7 more than the number of nickels, and the number of quarters is twice the number of dimes. If the total value of the coins is no greater than $7.20, what is the greatest possible number of nickels in the bank?

34. Rhoda is two-thirds as old as her sister Alice. Five years from now, the sum of their ages will be less than 60. What is the largest possible integral value for each sister's present age?

35. Four years ago, Bill was $1\frac{1}{4}$ times as old as his cousin Mary. The difference between their present ages is at least 3. What is the smallest possible integral value for each cousin's present age?

36. Mr. Drew invested a sum of money at $7\frac{1}{2}\%$ interest. He invested a second sum, which was \$200 less than the first, at 7% interest. If the total annual interest from these two investments is at least \$160, what is the smallest amount he could have invested at $7\frac{1}{2}\%$?

37. Mr. Lehtimaki wanted to sell his house. He advertised an asking price, but knew that he would accept, as a minimum, nine-tenths of the asking price. Mrs. Patel offered to buy the house, but her maximum offer was seven-eighths of the asking price. If the difference between the seller's lowest acceptance price and the buyer's maximum offer was at least \$3,000, find:

a. the minimum asking price for the house;

b. the minimum amount Mr. Lehtimaki, the seller, would accept;

c. the maximum amount offered by Mrs. Patel, the buyer.

38. When packing his books to move, Philip put the same number of books in each of 12 boxes. Once packed, the boxes were too heavy to lift so Philip removed one-fifth of the books from each box. If at least 100 books in total remain in the boxes, what is the minimum number of books that Philip originally packed in each box?

14-8 SOLVING FRACTIONAL EQUATIONS

An equation is called an **algebraic equation** when a variable appears in at least one of its sides. An algebraic equation is a **fractional equation** when a variable appears in the denominator of one, or more than one, of its terms. For example,

$$\frac{1}{3} + \frac{1}{x} = \frac{1}{2} \qquad \frac{2}{3d} + \frac{1}{3} = \frac{11}{6d} - \frac{1}{4} \qquad \frac{a^2+1}{a-1} + \frac{a}{2} = a + 2 \qquad \frac{1}{y^2+2y-3} + \frac{1}{y-2} = 3$$

are all fractional equations. To simplify such an equation, clear it of fractions by multiplying both sides by the least common denominator of all fractions in the equation. Then, solve the simpler equation. As is true of all algebraic fractions, a fractional equation has meaning only when values of the variable do not lead to a denominator of 0.

KEEP IN MIND When both sides of an equation are multiplied by a variable expression that may represent 0, the resulting equation may not be equivalent to the given equation. Such equations will yield **extraneous solutions**, which are solutions that satisfy the *derived* equation but not the *given* equation. Each solution, therefore, must be checked in the original equation.

EXAMPLE 1

Solve and check: $\frac{1}{3} + \frac{1}{x} = \frac{1}{2}$

Solution Multiply both sides of the equation by the least common denominator, $6x$.

$$
\begin{aligned}
\tfrac{1}{3} + \tfrac{1}{x} &= \tfrac{1}{2} \\
6x\left(\tfrac{1}{3} + \tfrac{1}{x}\right) &= 6x\left(\tfrac{1}{2}\right) \\
6x\left(\tfrac{1}{3}\right) + 6x\left(\tfrac{1}{x}\right) &= 6x\left(\tfrac{1}{2}\right) \\
2x + 6 &= 3x \\
6 &= x
\end{aligned}
$$

Check

$$
\begin{aligned}
\tfrac{1}{3} + \tfrac{1}{x} &= \tfrac{1}{2} \\
\tfrac{1}{3} + \tfrac{1}{\mathbf{6}} &\stackrel{?}{=} \tfrac{1}{2} \\
\tfrac{2}{6} + \tfrac{1}{6} &\stackrel{?}{=} \tfrac{1}{2} \\
\tfrac{3}{6} &\stackrel{?}{=} \tfrac{1}{2} \\
\tfrac{1}{2} &= \tfrac{1}{2} \checkmark
\end{aligned}
$$

Answer $x = 6$

EXAMPLE 2

Solve and check: $\frac{5x + 10}{x + 2} = 7$

Solution Multiply both sides of the equation by the least common denominator, $x + 2$.

$$
\begin{aligned}
\tfrac{5x + 10}{x + 2} &= 7 \\
(x + 2)\left(\tfrac{5x + 10}{x + 2}\right) &= (x + 2)(7) \\
5x + 10 &= 7x + 14 \\
-2x &= 4 \\
x &= -2
\end{aligned}
$$

Check

$$
\begin{aligned}
\tfrac{5x + 10}{x + 2} &= 7 \\
\tfrac{5(\mathbf{-2}) + 10}{-2 + 2} &\stackrel{?}{=} 7 \\
\tfrac{-10 + 10}{0} &\stackrel{?}{=} 7 \\
\tfrac{0}{0} &= 7 \text{ ✘}
\end{aligned}
$$

The only possible value of x is a value for which the equation has no meaning because it leads to a denominator of 0. Therefore, there is no solution for this equation.

Answer The solution set is the empty set, $\varnothing$ or $\{\ \}$.

EXAMPLE 3

Solve and check: $\frac{2}{x} = \frac{6 - x}{4}$

Solution

METHOD 1

Multiply both sides of the equation by the LCD, $4x$:

$$\begin{aligned} 4x\left(\tfrac{2}{x}\right) &= 4x\left(\tfrac{6-x}{4}\right) \\ 8 &= x(6-x) \\ 8 &= 6x - x^2 \\ x^2 - 6x + 8 &= 0 \\ (x-2)(x-4) &= 0 \end{aligned}$$

$$\begin{array}{r|r} x - 2 = 0 & x - 4 = 0 \\ x = 2 & x = 4 \end{array}$$

METHOD 2

Use the rule for proportion: *the product of the means equals the product of the extremes.*

$$\begin{aligned} \tfrac{2}{x} &= \tfrac{6-x}{4} \\ x(6-x) &= 8 \\ 6x - x^2 &= 8 \\ -x^2 + 6x - 8 &= 0 \\ x^2 - 6x + 8 &= 0 \\ (x-2)(x-4) &= 0 \end{aligned}$$

$$\begin{array}{r|r} x - 2 = 0 & x - 4 = 0 \\ x = 2 & x = 4 \end{array}$$

Check

$$\begin{array}{l|l} x = 2 & x = 4 \\ \frac{2}{x} = \frac{6-x}{4} & \frac{2}{x} = \frac{6-x}{4} \\ \frac{2}{\mathbf{2}} = \frac{6-\mathbf{2}}{4} & \frac{2}{\mathbf{4}} = \frac{6-\mathbf{4}}{4} \\ 1 = 1 \checkmark & \frac{1}{2} = \frac{1}{2} \checkmark \end{array}$$

Answer $x = 2$ or $x = 4$

EXAMPLE 4

Solve and check: $\frac{1}{x+2} + \frac{1}{x-1} = \frac{1}{2(x-1)}$

Solution Multiply both sides of the equation by the LCD, $2(x + 2)(x - 1)$:

$$\begin{aligned} 2(x+2)(x-1)\left(\frac{1}{x+2} + \frac{1}{x-1}\right) &= \left(\frac{1}{2(x-1)}\right)2(x+2)(x-1) \\ \frac{2(x+2)(x-1)}{x+2} + \frac{2(x+2)(x-1)}{x-1} &= \frac{2(x+2)(x-1)}{2(x-1)} \\ \frac{2\cancel{(x+2)}(x-1)}{\cancel{x+2}} + \frac{2(x+2)\cancel{(x-1)}}{\cancel{x-1}} &= \frac{\cancel{2}(x+2)\cancel{(x-1)}}{\cancel{2(x-1)}} \\ 2(x-1) + 2(x+2) &= x + 2 \\ 4x + 2 &= x + 2 \\ 3x &= 0 \\ x &= 0 \end{aligned}$$

Check

$$\begin{aligned} \frac{1}{\mathbf{0}+2} + \frac{1}{\mathbf{0}-1} &\stackrel{?}{=} \frac{1}{2(\mathbf{0}-1)} \\ \frac{1}{2} + \frac{1}{-1} &\stackrel{?}{=} \frac{1}{2(-1)} \\ -\frac{1}{2} &= -\frac{1}{2} \checkmark \end{aligned}$$

Answer $x = 0$

EXERCISES

Writing About Mathematics

1. Nathan said that the solution set of $\frac{2}{r-5} = \frac{10}{5r-25}$ is the set of all real numbers. Do you agree with Nathan? Explain why or why not.

2. Pam multiplied each side of the equation $\frac{y+5}{y^2-25} = \frac{3}{y+5}$ by $(y+5)(y-5)$ to obtain the equation $y + 5 = 3y - 15$, which has as its solution $y = 10$. Pru said that the equation $\frac{y+5}{y^2-25} = \frac{3}{y+5}$ is a proportion and can be solved by writing the product of the means equal to the product of the extremes. She obtained the equation $3(y^2 - 25) = (y + 5)^2$, which has as its solution 10 and -5. Both girls used a correct method of solution. Explain the difference in their answers.

Developing Skills

In 3–6, explain why each fractional equation has no solution.

3. $\frac{6x}{x} = 3$ **4.** $\frac{4a+4}{a+1} = 5$ **5.** $\frac{2}{x} = 4 + \frac{2}{x}$ **6.** $\frac{x}{x-1} + 2 = \frac{1}{x-1}$

In 7–45, solve each equation, and check.

7. $\frac{10}{x} = 5$ **8.** $\frac{15}{y} = 3$ **9.** $\frac{3}{2x} = \frac{1}{2}$

10. $\frac{15}{4x} = \frac{1}{8}$ **11.** $\frac{10}{x} + \frac{8}{x} = 9$ **12.** $\frac{15}{y} - \frac{3}{y} = 4$

13. $\frac{9}{2x} = \frac{7}{2x} + 2$ **14.** $\frac{30}{x} = 7 + \frac{18}{2x}$ **15.** $\frac{y-2}{2y} = \frac{3}{8}$

16. $\frac{x-5}{x} + \frac{3}{x} = \frac{2}{3}$ **17.** $\frac{y+9}{2y} + 3 = \frac{15}{y}$ **18.** $\frac{1}{3a} + \frac{5}{12} = \frac{2}{a}$

19. $\frac{b-6}{b} - \frac{1}{6} = \frac{4}{b}$ **20.** $\frac{7}{8} - \frac{x-3}{x} = \frac{1}{4}$ **21.** $\frac{5+x}{2x} - 1 = \frac{x+1}{x}$

22. $\frac{2+x}{6x} = \frac{3}{5x} + \frac{1}{30}$ **23.** $\frac{a}{a+2} - \frac{a-2}{a} = \frac{1}{a}$ **24.** $\frac{x+1}{2x} + \frac{2x+1}{3x} = 1$

25. $\frac{6}{3x-1} = \frac{3}{4}$ **26.** $\frac{2}{3x-4} = \frac{1}{4}$ **27.** $\frac{5x}{x+1} = 4$

28. $\frac{3}{5-3a} = \frac{1}{2}$ **29.** $\frac{4z}{7+5z} = \frac{1}{3}$ **30.** $\frac{1-r}{1+r} = \frac{2}{3}$

31. $\frac{3}{y} = \frac{2}{5-y}$ **32.** $\frac{5}{a} = \frac{7}{a-4}$ **33.** $\frac{2}{m} = \frac{5}{3m-1}$

34. $\frac{2}{a-4} = \frac{5}{a-1}$ **35.** $\frac{12}{2-x} = \frac{15}{7+x}$ **36.** $\frac{12y}{8y+5} = \frac{2}{3}$

37. $\frac{y}{y+1} - \frac{1}{y} = 1$ **38.** $\frac{x}{x^2-9} = \frac{1}{x+3}$ **39.** $\frac{2}{x} = \frac{x-3}{2}$

40. $\frac{a-1}{6} = \frac{1}{a}$ **41.** $\frac{1}{b} = \frac{b-1}{2}$ **42.** $\frac{3}{2b+1} = \frac{b}{2}$

43. $\frac{1}{b-1} + \frac{1}{6} = \frac{b+2}{12}$ **44.** $\frac{1}{x} - \frac{1}{9} = \frac{x-5}{18}$ **45.** $\frac{1}{x+2} + \frac{1}{2x+4} = -\frac{3}{2}x$

In 46–49, solve each equation for x in terms of the other variables.

46. $\frac{t}{x} - k = 0$ **47.** $\frac{t}{x} - k = 5k$ **48.** $\frac{a+b}{x} = c$ **49.** $\frac{d}{x} = \frac{d-1}{x-e}$

50. If $x = \frac{by}{c}$, $y = \frac{c^2}{a}$, $b = \frac{a}{c}$, $a \neq 0$, and $c \neq 0$, is it possible to know the numerical value of x without knowing numerical values of a, b, c, and y? Explain your answer

Applying Skills

51. If 24 is divided by a number, the result is 6. Find the number.

52. If 10 is divided by a number, the result is 30. Find the number.

53. The sum of 20 divided by a number, and 7 divided by the same number, is 9. Find the number.

54. When the reciprocal of a number is decreased by 2, the result is 5. Find the number.

55. The numerator of a fraction is 8 less than the denominator of the fraction. The value of the fraction is $\frac{3}{5}$. Find the fraction.

56. The numerator and denominator of a fraction are in the ratio 3 : 4. When the numerator is decreased by 4 and the denominator is increased by 2, the value of the new fraction, in simplest form, is $\frac{1}{2}$. Find the original fraction.

57. The ratio of boys to girls in the chess club is 4 to 5. After 2 boys leave the club and 2 girls join, the ratio is 1 to 2. How many members are in the club?

58. The length of Emily's rectangular garden is 4 feet greater than its width. The width of Sarah's rectangular garden is equal to the length of Emily's and its length is 18 feet. The two gardens are similar rectangles, that is, the ratio of the length to the width of Emily's garden equals the ratio of the length to the width of Sarah's garden. Find the possible dimensions of each garden. (Two answers are possible.)

CHAPTER SUMMARY

An **algebraic fraction** is the quotient of two algebraic expressions. If the algebraic expressions are polynomials, the fraction is called a **rational expression** or a **fractional expression**. An algebraic fraction is defined only if values of the variables do not result in a denominator of 0.

Fractions that are equal in value are called **equivalent fractions**. A fraction is **reduced to lowest terms** when an equivalent fraction is found such that its numerator and denominator have no common factor other than 1 or -1. This fraction is considered a **lowest terms fraction**.

Operations with algebraic fractions follow the same rules as operations with arithmetic fractions:

Multiplication

$$\frac{a}{x} \cdot \frac{b}{y} = \frac{ab}{xy}\ (x \neq 0, y \neq 0)$$

Division

$$\frac{a}{x} \div \frac{b}{y} = \frac{a}{x} \cdot \frac{y}{b} = \frac{ay}{bx}\ (x \neq 0, y \neq 0, b \neq 0)$$

Addition/subtraction with the same denominator

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}\ (c \neq 0), \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}(c \neq 0)$$

Addition/subtraction with different denominators

(first, obtain the common denominator):

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}\ (b \neq 0, d \neq 0)$$

A **fractional equation** is an equation in which a variable appears in the denominator of one or more than one of its terms. To simplify a fractional equation, or any equation or inequality containing fractional coefficients, multiply both sides by the **least common denominator (LCD)** to eliminate the fractions. Then solve the simpler equation or inequality and check for **extraneous solutions**.

VOCABULARY

14-1 Algebraic fraction • Fractional expression • Rational expression

14-2 Reduced to lowest terms • Lowest terms fraction • Equivalent fractions • Division property of a fraction • Cancellation • Multiplication property of a fraction

14-3 Cancellation method

14-5 Common denominator • Least common denominator

14-8 Algebraic equation • Fractional equation • Extraneous solution

REVIEW EXERCISES

1. Explain the difference between an algebraic fraction and a fractional expression.
2. What fractional part of 1 centimeter is x millimeters?
3. For what value of y is the fraction $\frac{y-1}{y-4}$ undefined?
4. Factor completely: $12x^3 - 27x$

In 5–8, reduce each fraction to lowest terms.

5. $\frac{8bg}{12bg}$ **6.** $\frac{14d}{7d^2}$ **7.** $\frac{5x^2 - 60}{5}$ **8.** $\frac{8y^2 - 12y}{8y}$

In 9–23, in each case, perform the indicated operation and express the answer in lowest terms.

9. $\frac{3x^2}{4} \cdot \frac{8}{9x}$ **10.** $\frac{2x - 2}{3y} \cdot \frac{3xy}{2x}$ **11.** $6c^2 \div \frac{c}{2}$

12. $\frac{3a}{7b} \div \frac{18a}{35}$ **13.** $\frac{5m}{6} - \frac{m}{6}$ **14.** $\frac{9}{k} - \frac{3}{k} + \frac{4}{k}$

15. $\frac{ax}{3} + \frac{ax}{4}$ **16.** $\frac{5}{xy} - \frac{2}{yz}$ **17.** $\frac{4x + 5}{3x} + \frac{2x + 1}{2x}$

18. $\frac{x^2 - 5x}{x^2} \cdot \frac{x}{2x - 10}$ **19.** $\frac{2a}{a + b} + \frac{2b}{a + b}$ **20.** $\frac{y + 7}{5} - \frac{y + 3}{4}$

21. $\frac{x^2 - 25}{12} \div \frac{x^2 - 10x + 25}{4}$ **22.** $\frac{c - 3}{12} + \frac{c + 3}{8}$ **23.** $\frac{3a - 9a^2}{a} \div (1 - 9a^2)$

24. If the sides of a triangle are represented by $\frac{b}{2}$, $\frac{5b}{6}$, and $\frac{2b}{3}$, express the perimeter of the triangle in simplest form.

25. If $a = 2$, $b = 3$, and $c = 4$, what is the sum of $\frac{b}{a} + \frac{a}{c}$?

In 26–31, solve each equation and check.

26. $\frac{k}{20} = \frac{3}{4}$ **27.** $\frac{x - 3}{10} = \frac{4}{5}$ **28.** $\frac{y}{2} - \frac{y}{6} = 4$

29. $\frac{6}{m} = \frac{20}{m} - 2$ **30.** $\frac{2t}{5} - \frac{t - 2}{10} = 2$ **31.** $\frac{1}{a - 1} - \frac{1}{a} = \frac{1}{20}$

In 32–34, solve each equation for r in terms of the other variables.

32. $\frac{S}{h} = 2\pi r$ **33.** $\frac{c}{2r} = \pi$ **34.** $\frac{a}{r} - n = 0$

35. Mr. Vroman deposited a sum of money in the bank. After a few years, he found that the interest equaled one-fourth of his original deposit and he had a total sum, deposit plus interest, of $2,400 in the bank. What was the original deposit?

36. One-third of the result obtained by adding 5 to a certain number is equal to one-half of the result obtained when 5 is subtracted from the number. Find the number.

37. Of the total number of points scored by the winning team in a basketball game, one-fifth was scored in the first quarter, one-sixth was scored in the second quarter, one-third was scored in the third quarter, and 27 was scored in the fourth quarter. How many points did the winning team score?

38. Ross drove 300 miles at r miles per hour and 360 miles at $r + 10$ miles per hour. If the time needed to drive 300 miles was equal to the time needed to drive 360 miles, find the rates at which Ross drove. (Express the time needed for each part of the trip as $t = \frac{d}{r}$.)

39. The total cost, T, of n items that cost a dollars each is given by the equation $T = na$.

a. Solve the equation $T = na$ for n in terms of T and a.

b. Use your answer to **a** to express n_1, the number of cans of soda that cost \$12.00 if each can of soda costs a dollars.

c. Use your answer to **a** to express n_2, the number of cans of soda that cost \$15.00 if each can of soda costs a dollars.

d. If the number of cans of soda purchased for \$12.00 is 4 less than the number purchased for \$15.00, find the cost of a can of soda and the number of cans of soda purchased.

40. The cost of two cups of coffee and a bagel is \$1.75. The cost of four cups of coffee and three bagels is \$4.25. What is the cost of a cup of coffee and the cost of a bagel?

41. A piggybank contains nickels, dimes, and quarters. The number of nickels is 4 more than the number of dimes, and the number of quarters is 3 times the number of nickels. If the total value of the coins is no greater than \$8.60, what is the greatest possible number of dimes in the bank?

Exploration

Some rational numbers can be written as terminating decimals and others as infinitely repeating decimals.

(1) Write each of the following fractions as a decimal:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{16}, \frac{1}{20}, \frac{1}{25}, \frac{1}{50}, \frac{1}{100}$$

(2) What do you observe about the decimals written in (1)?

(3) Write each denominator in factored form.

(4) What do you observe about the factors of the denominators?

(5) Write each of the following fractions as a decimal:

$$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{11}, \frac{1}{12}, \frac{1}{15}, \frac{1}{18}, \frac{1}{22}, \frac{1}{24}, \frac{1}{30}$$

What do you observe about the decimals written in (5)?

(7) Write each denominator in factored form.

(8) What do you observe about the factors of the denominators?

(9) Write a statement about terminating and infinitely repeating decimals based on your observations.

CUMULATIVE REVIEW CHAPTERS 1–14

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. The product of $3a^2$ and $5a^5$ is

(1) $15a^{10}$ (2) $15a^7$ (3) $8a^{10}$ (4) $8a^7$

2. In the coordinate plane, the point whose coordinates are $(-2, 1)$ is in quadrant

(1) I (2) II (3) III (4) IV

3. In decimal notation, 3.75×10^{-2} is

(1) 0.0375 (2) 0.00375 (3) 37.5 (4) 375

4. The slope of the line whose equation is $3x - y = 5$ is

(1) 5 (2) -5 (3) 3 (4) -3

5. Which of the following is an irrational number?

(1) 1.3 (2) $\frac{2}{3}$ (3) $\sqrt{9}$ (4) $\sqrt{5}$

6. The factors of $x^2 - 7x - 18$ are

(1) 9 and -2 (3) $(x - 9)$ and $(x + 2)$
(2) -9 and 2 (4) $(x + 9)$ and $(x - 2)$

7. The dimensions of a rectangular box are 8 by 5 by 9. The surface area is

(1) 360 cubic units (3) 157 square units
(2) 360 square units (4) 314 square units

8. The length of one leg of a right triangle is 8 and the length of the hypotenuse is 12. The length of the other leg is

(1) 4 (2) $4\sqrt{5}$ (3) $4\sqrt{13}$ (4) 80

9. The solution set of $\frac{a^2}{a+1} - 1 = \frac{1}{a+1}$ is

(1) $\{-1, 2\}$ (2) $\{2\}$ (3) $\{0, 1\}$ (4) $\{1, -2\}$

10. In the last n times a baseball player was up to bat, he got 3 hits and struck out the rest of the times. The ratio of hits to strike-outs is

(1) $\frac{3}{n}$ (2) $\frac{n-3}{n}$ (3) $\frac{3}{n-3}$ (4) $\frac{n-3}{3}$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. Mrs. Kniger bought some stock on May 1 for \$3,500. By June 1, the value of the stock was \$3,640. What was the percent of increase of the cost of the stock?

12. A furlong is one-eighth of a mile. A horse ran 10 furlongs in 2.5 minutes. What was the speed of the horse in feet per second?

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. Two cans of soda and an order of fries cost \$2.60. One can of soda and two orders of fries cost \$2.80. What is the cost of a can of soda and of an order of fries?

14. **a.** Draw the graph of $y = x^2 - 2x$.

b. From the graph, determine the solution set of the equation $x^2 - 2x = 3$.

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. If the measure of the smallest angle of a right triangle is 32° and the length of the shortest side is 36.5 centimeters, find the length of the hypotenuse of the triangle to the *nearest tenth of a centimeter*.

16. The area of a garden is 120 square feet. The length of the garden is 1 foot less than twice the width. What are the dimensions of the garden?